

Course ID: MATH101 Course Name: Advanced Algebra Test Name: Problem-Solving and Proofs Student ID: 20240001 Student Name: John Doe

To determine a , b , and c , we substitute the given points into the quadratic equation:

$$\begin{aligned} 1. f(1)=4 &\Rightarrow a(1)^2+b(1)+c=4 \Rightarrow a(1)^2 + b(1) + c = 4 \\ 4f(1)=4 &\Rightarrow a(1)^2+b(1)+c=4 \\ a+b+c &= 4a + b + c = 4a+b+c=4 \end{aligned}$$

$$\begin{aligned} 2. f(2)=7 &\Rightarrow a(2)^2+b(2)+c=7 \Rightarrow a(2)^2 + b(2) + c = 7 \\ 7f(2)=7 &\Rightarrow a(2)^2+b(2)+c=7 \\ 4a+2b+c &= 7 \Rightarrow 4a + 2b + c = 7 \end{aligned}$$

$$\begin{aligned} 3. f(3)=12 &\Rightarrow a(3)^2+b(3)+c=12 \Rightarrow a(3)^2 + b(3) + c = 12 \\ 12f(3)=12 &\Rightarrow a(3)^2+b(3)+c=12 \\ 9a+3b+c &= 12 \Rightarrow 9a + 3b + c = 12 \end{aligned}$$

We now solve this system of equations:

- Subtracting the first equation from the second:
 $(4a+2b+c)-(a+b+c)=7-4 \Rightarrow 3a+b=3$

- Subtracting the second equation from the third:
 $(9a+3b+c)-(4a+2b+c)=12-7 \Rightarrow 5a+b=5$

- Subtracting the two new equations:
 $(5a+b)-(3a+b)=5-3 \Rightarrow 2a=2 \Rightarrow a=1$

Substituting $a=1$ into $3a+b=3$:
 $3(1)+b=3 \Rightarrow b=0$
 Substituting $a=1$ and $b=0$ into $a+b+c=4$:
 $1+0+c=4 \Rightarrow c=3$

Thus, the function is:
 $f(x)=x^2+3$